REGIME-DEPENDENT EFFECTS OF UNCERTAINTY SHOCKS: A STRUCTURAL INTERPRETATION

STÉPHANE LHUISSIER* AND FABIEN TRAPIER**

ABSTRACT. Using a Markov-switching VAR, we show that the effects of uncertainty shocks on output are four times higher in a regime of economic distress than in a tranquil regime. We then provide a structural interpretation of these facts. To do so, we develop a business cycle model, in which agents are aware of the possibility of regime changes when forming expectations. The model is estimated using a Bayesian minimum distance estimator that minimizes, over the set of structural parameters, the distance between the regime-switching VAR-based impulse response functions and those implied by the model. Our results point to changes in the degree of financial frictions. We discuss the implications of this structural interpretation and show that the expectation effect of regime switching in financial conditions is an important component of the financial accelerator mechanism. If agents hold pessimistic expectations about future financial conditions, then shocks are amplified and transmitted more rapidly to the economy.

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I. Introduction

It has been well documented that higher uncertainty reduces aggregate activity, leading to higher unemployment, and lower investment and output.\(^1\) Recent empirical studies have also emphasized highly nonlinear effects, depending on the state of the economy; adverse effects of uncertainty shocks are greater in periods of economic distress than in tranquil periods.\(^2\) However, little is known about the structural factors in accounting for these changes as inference of nonlinear relationships presents econometric challenges within a quantitative general equilibrium framework.

The objective of this paper is to fill part of this gap by exploring, through a novel econometric estimation, potential changes in the underlying structure of the economy that could explain such a nonlinearity. Disentangling the causes is important for understanding the extent to which economic activity can respond to future uncertainty shocks as well as the role that policy can play in order to mitigate those adverse effects.

We first reproduce the empirical evidence of highly nonlinear effects within a Markov-switching Structural Vector Autoregression (MS-SVAR) framework. We use U.S. quarterly data and include GDP growth, a measure of uncertainty (i.e., the VIX index), and a credit spread. The model identifies two distinct regimes. The first was seen in nearly all the years of episodes of high inflationary pressure in the 1970s and 1980s, during serious turbulence that marked 2001-2003 period (including the 9/11 terrorist attacks, Dot-com bubble, and corporate scandals), and during the global financial crisis. The second covers periods of tranquility. We show that, under the first regime, the adverse output effects of an increase in uncertainty appear to be four times higher than under the second regime.

We then focus on the potential explanations for this regime-dependent evidence by estimating the key macroeconomic and financial parameters of a Markov-switching Dynamic Stochastic General Equilibrium (MS-DSGE) model with financial frictions, as in Bernanke, Gertler, and Gilchrist (1999), and uncertainty shocks along the line of Christiano, Motto, and

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Our empirical approach is analogous to the impulse response matching approach used by Rotemberg and Woodford (1997) and Christiano, Eichenbaum, and Evans (2005), except that we are estimating the parameters to fit our regime-dependent impulse responses from a MS-SVAR, as opposed to impulse responses from a constant-parameters SVAR. To the best of our knowledge, our paper represents the first attempt to estimate a medium-scale Markov-switching DSGE model by matching the MS-SVAR-implied impulse responses to those produced by the MS-DSGE model. We believe our MS-SVAR-implied impulse responses approach is a promising tool to infer MS-DSGE models, and can be seen as an alternative to the full Bayesian approach implemented notably by Liu, Waggoner, and Zha (2011) and Bianchi (2013).

Our estimates imply that the differences in impulse responses across regimes result mainly from changes in the degree of financial frictions. In particular, lenders pay a much higher monitoring cost in the distress regime than in the tranquil regime, implying therefore a more powerful financial accelerator; i.e., linkages between the quality of borrowers’ balance sheets and their access to external finance are strengthened. It then becomes straightforward to understand why the response of the economy to uncertainty shocks differs across regimes. When uncertainty rises, banks protect themselves by raising the interest rate charged on loans to firms (i.e., external finance premium), as there are more low-productivity firms — and also more high-productivity firms, but this does not benefit to banks — and thus more default risks. It follows a decline in demand for capital, and so in investment spending and economic activity. In distress periods, the premium becomes much more sensitive to changes in the firm’s balance sheet, causing firms to make larger cut to their investment projects, and therefore, implying a larger and longer-lasting decline in economic activity.

The key insight of our MS-DSGE model is that variations in the MS-SVAR dynamics of the effects of uncertainty shocks have important effects on rational agents’ expectation formation of the MS-DSGE model. Our estimates lie in the fact that agents are aware of the possibility of regime switches in the dynamics. That is, our MS-SVAR-based impulse response matching approach takes into account the fact that all agents of the MS-DSGE

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model know the probabilities assigned by the Markov-switching process of the MS-SVAR model and use them when forming expectations.

Under these circumstances, in any given regime, agents anticipate that uncertainty shocks may be accompanied by a switch to the other regime, altering considerably the macroeconomic outcomes. We consider how these expectation effects, using the terminology of Liu, Waggoner, and Zha (2009), on a particular regime may affect equilibrium in the other regime. In tranquil periods, characterized by a small degree of agency problems, agents may expect that the economy will move to the distress regime. This over-pessimistic behavior, anticipating that the possibility that the agency problems may become more severe in the future, will lead to amplify the contractionary effects of uncertainty shocks on aggregate activity. Conversely, an over-optimistic behavior dampens these negative effects. As a result, the expectation effects of regime shifts in financial conditions are part of the financial accelerator mechanism.

This paper proceeds as follows. Section II relates our contributions to the literature. To illustrate the possibility of nonlinearity between uncertainty and the macroeconomy, Section III provides empirical insights into how different the impact of uncertainty on aggregate activity is between distress and non-stress periods. Section IV interprets these differences in terms of an estimated DSGE model with financial frictions, in which agents form expectations on possible changes on the economy, and investigates the expectation effects of regime switching in the degree of financial frictions. Section V concludes.

II. Literature review

This paper is related to an increasing literature that examines how uncertainty manifests itself and what their effects are on the rest of the economy.

Focusing on the United States, Bloom (2009), Stock and Watson (2012), Bekaert, Hoerova, and Duca (2013), Glover and Levine (2015), Leduc and Liu (2016), Basu and Bundick (2017), Creal and Wu (2017) and Ferrara and Guérin (2018), employ the “constant-parameters” approach to quantify the role of uncertainty on business cycle fluctuations. In particular, all studies adopt linear SVARs and find a significant and long-lasting decrease of aggregate

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4Liu, Waggoner, and Zha (2009) originally defined the expectation effects for monetary policy as "the difference between equilibrium outcome from a model that ignores probabilistic shifts in future policy regime and that from a model that takes into account such expected changes in regime".
activity after a positive uncertainty shock. Empirical studies have been rapidly interested in their time-varying effects as events of high uncertainty did not always seem to spill over to the economy.\(^5\)

Mumtaz and Theodoridis (2016) extend the standard approach by allowing time-varying parameters in SVARs. They emphasize the importance of taking into account shifts in the generation of uncertainty shocks. They show, in particular, the impact of uncertainty shocks on aggregate activity has declined over time. However, the limitation of this paper to study episodes of distress, as considered herein, lies in the methodology itself — a model with smooth and drifting coefficients seems to be less suited for capturing rapid shifts in the behavior of the data as observed during distress periods. Economic or financial crises are well-known for hitting the economy instantaneously, which favors models with abrupt changes like Markov-switching models. Therefore, we follow Sims and Zha (2006) and estimate a MS-SVAR with Bayesian methods. Hubrich and Tetlow (2015) and Lhuissier (2017) also consider a MS-SVAR framework to capture regime switching in macroeconomic time series in distress periods.

Employing an alternative regime-switching method (i.e., a threshold VAR model), Caggiano, Castelnuovo, and Groshenny (2014), Caggiano, Castelnuovo, and Nodari (2017), and Alessandri and Mumtaz (2018) show that the real effects of uncertainty shocks strongly depend on the state of the economy. In particular, Alessandri and Mumtaz (2018) show that the effects depend on the state of financial markets and estimate that the impact on output is five times larger in periods of financial stress than in tranquil periods, while Caggiano, Castelnuovo, and Groshenny (2014) and Caggiano, Castelnuovo, and Nodari (2017) capture recession and expansion phases and show that uncertainty shocks are substantially more costly under recessions than under expansions. Our approach clearly differs since we assign probabilities to events and, therefore, we avoid to make the assumption that the probability of a regime switch is either one or zero. Moreover, estimating these probabilities is essential to analyze the importance of expectation effects of regime shifts in equilibrium dynamics of our MS-DSGE model, and therefore, in the transmission mechanism of uncertainty shocks to the aggregate economy.

\(^5\)Bloom (2009) documents a variety of events that generate significant uncertainty about the future, but they are not always associated with large decline in output.
Our analysis is related to a growing body of evidence which documents the interactions between uncertainty and financial conditions within an equilibrium business cycle framework — notable examples are Christiano, Motto, and Rostagno (2014), Gilchrist, Sim, and Zakrajšek (2014), Bloom, Alfaro, and Lin (2019), Brand, Isoré, and Tripier (2019), and Arellano, Bai, and Kehoe (forthcoming). More specifically, our framework closely follows Christiano, Motto, and Rostagno (2014), who investigate the real role of uncertainty shocks in the context of the financial accelerator model initially developed by Bernanke, Gertler, and Gilchrist (1999). Note, however, that the severity of agency problems (i.e., monitoring costs) remains unchanged over time within their framework. Levin, Natalucci, and Zakrajšek (2004), and more recently, Lindé, Smets, and Wouters (2016) and Fuentes-Albero (2018) make it time-varying without, however, investigating the macroeconomic implications of uncertainty shocks, and the role of expectation effects of regime shifts in financial frictions in shaping the macroeconomic outcomes.

Our paper is also related to an increasing literature investigating the importance of expectation effects in regime shifts in a Markov-switching framework. This concept was originally defined by Liu, Waggoner, and Zha (2009) in the context of regime changes in monetary policy, and then have been extensively studied thereafter. Bianchi (2013) considers “beliefs counterfactuals” to quantify the importance of expectation effects in business cycle fluctuations. Foerster (2016) distinguishes the expectation effects of regime switching in the inflation target from those in the inflation response. Bianchi and Ilut (2017) allow for monetary/fiscal policy mix changes. We extend this concept and apply it for regime shifts in the degree of financial frictions. Interestingly, the expectation effects embedded in our model share some features with the anticipation effect described by He and Krishnamurthy (forthcoming) in the context of a model with occasionally binding financial constraints. In their model, financial constraints have effects on the equilibrium even when they are not binding (which corresponds to the tranquil regime in our model) if agents anticipate that they may bind in the future (which corresponds to the realization of the stress regime in our model).

From a methodological standpoint, this paper is related to an increasing literature dealing with estimation and simulation of DSGE models in which stochastic volatilities and structural

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6It is worth mentioning that Bianchi and Melosi (2016) extend this framework to consider Bayesian learning by rational agents.
parameters are allowed to follow Markov-switching processes. This literature includes, among others, Liu, Waggoner, and Zha (2011), Bianchi (2013), Davig and Doh (2014), Lhuissier and Zabelina (2015), Bianchi and Melosi (2017), and Lhuissier (2018). The standard approach for inference of MS-DSGE models employed by all of these papers is to build the state-space representation of the MS-DSGE models adapted from the the standard Kim and Nelson (1999)'s filter. In contrast, our approach dispenses with such a filter as inference is directly done by minimizing the gap between theoretical and empirical impulse response functions.

III. Evidence of time variation in the effects of uncertainty shocks

This section documents changes in the effects of uncertainty shocks on aggregate activity over time by employing a Markov-switching framework.

III.1. Markov-switching Structural Bayesian VARs. Following Hamilton (1989), Sims and Zha (2006), and Sims, Waggoner, and Zha (2008), we employ a Markov-switching Bayesian structural VAR model of the following form:

\[
y'_{t-1}A(s^c_t) = \rho \sum_{i=1}^{\rho} y'_{t-i}A_i(s^c_t) + C(s^c_t) + \varepsilon'_t \Xi^{-1}(s^y_t), \quad t = 1, \ldots, T, \tag{1}
\]

where \( y_t \) is defined as \( y_t \equiv [gdp_t, vix_t, sp_t]' \); \( gdp_t \) is the logarithm of U.S. real GDP; \( vix_t \) is the VIX index, a proxy for uncertainty; and \( sp_t \) is the BAA-AAA credit spread. Data sources are presented in Appendix A. The overall sample period is 1962:Q3 to 2018:Q2. We set the lag order to \( \rho = 2 \). Our parsimonious specification is justified by the fact that it becomes quickly challenging to estimate Bayesian MS-SVAR models as the number of observables and lags grows. Note also that this is in line with the literature that allows for time-varying parameters in VARs (e.g., Primiceri, 2005; Cogley and Sargent, 2005; Bianchi and Melosi, 2017).

We assume a two-regimes process governing equation coefficients and constants \( (s_t^c) \), and a three-regimes process governing disturbance variances \( (s_t^y) \). The regimes evolve according to two transition matrices as follows:

\[
Q^c = \begin{bmatrix}
q_{1,1}^c & q_{1,2}^c \\
q_{2,1}^c & q_{2,2}^c
\end{bmatrix}, \quad \text{and} \quad Q^y = \begin{bmatrix}
q_{1,1}^y & (1 - q_{2,2}^y)/2 & 0 \\
1 - q_{1,1}^y & q_{2,2}^y & 1 - q_{3,3}^y \\
0 & (1 - q_{2,2}^y)/2 & q_{3,3}^y
\end{bmatrix}. \tag{2}
\]
The restricted transition matrix \( Q \) implies that when we are in regime \( j \), we can only move to regime \( j-1 \) or \( j+1 \). Sims, Waggoner, and Zha (2008) argue that such a restriction tends to fit the macroeconomic data better.

We assume that \( \varepsilon_t \) follows the following distribution:

\[
p(\varepsilon_t) = \text{normal}(\varepsilon_t|0_n, I_n),
\]

where \( 0_n \) denotes an \( n \times 1 \) vector of zeros, \( I_n \) denotes the \( n \times n \) identity matrix, and \( \text{normal}(x|\mu, \Sigma) \) denotes the multivariate normal distribution of \( x \) with mean \( \mu \) and variance \( \Sigma \). Finally, \( T \) is the sample size; \( A(s_t) \) is a \( n \)-dimensional invertible matrix under the regime \( s_t \); \( A_i(s_t) \) is a \( n \)-dimensional matrix that contains the coefficients at the lag \( i \) and the regime \( s_t \); \( C(s_t) \) contains the constant terms; and \( \Xi(s_t) \) is a \( n \)-dimensional diagonal matrix.

Following Sims and Zha (1998), we exploit the idea of a Litterman’s random-walk prior to structural-form parameters.\(^7\) Appendix B provides the details techniques for the Sims and Zha (1998) prior.

Finally, the prior duration of each regime is about five quarters. We have also used other prior duration and the main conclusions remain unchanged.

III.2. Identification. We identify uncertainty shocks by combining two kinds of restriction. The first is based on traditional sign restrictions on the impulse response functions, as developed by Faust (1998), Canova and Nicolo (2002), and Uhlig (2005). We impose that an uncertainty shock induces a simultaneous rise in the VIX index and credit spread. The argument for this restriction is based on the idea that increases in financial uncertainty is frequently associated with significant increases in credit spreads, as shown in Stock and Watson (2012). We also assume that innovations to uncertainty cause a fall in output. This restriction is motivated by the large theoretical literature views that uncertainty has recessionary effects. See Bloom (2014) for a survey of the literature.

\(^7\)Regarding the Sims and Zha (1998) prior, the hyperparameters are defined as follows: \( \mu_1 = 1.00 \) (overall tightness of the random walk prior); \( \mu_2 = 1.00 \) (relative tightness of the random walk prior on the lagged parameters); \( \mu_3 = 0.1 \) (relative tightness of the random walk prior on the constant term); \( \mu_4 = 1.0 \) (erratic sampling effects on lag coefficients); \( \mu_5 = 0.0 \) (belief about unit roots); and \( \mu_6 = 0.0 \) (belief in cointegration relationships). To match the usual interpretation of Litterman’s prior on the reduced form, we drop the two kinds of true dummy observations (\( \mu_5 \) and \( \mu_6 \)) introduced by Sims and Zha (1998). See also Doan, Litterman, and Sims (1984) and Sims (1993).
The above restriction is not sufficient to guarantee pure uncertainty shocks due to the high degree of comovement between the uncertainty proxy and credit spread. It might be possible that shocks originating from financial sector are present into uncertainty shocks. The second kind of restriction allows us to completely disentangle between these two types of shock. We use a criterion that impose a restriction on forecasting error variance decompositions (FEVD). More specifically, uncertainty shock should at least explain 50 percent of variations in the VIX index. This kind of restriction is in line with Caldara, Fuentes-Albero, Gilchrist, and Zakrajek (2016) who identify uncertainty shocks as innovations explaining the maximum amount of variability in an uncertainty indicator to disentangle them from financial shocks.

By combining the appeal of FEVD-restrictions approach with the advantages of sign restrictions, we are able to isolate fluctuations in uncertainty and its effects on economic activity.

III.3. Empirical results. In this section, we report our main empirical results produced by the MS-SVAR model. First, we present, in Section III.3.1, the posterior distribution of the estimated model. We then report, in Section III.3.2, impulse responses of endogenous variables to uncertainty shock.

The results shown are based on 10 million draws with the Gibbs sampling procedure (see Appendix B for details). We discard the first 1,000,000 draws as burn-in, then keep every 100th draw.

III.3.1. Posterior distribution. In this section, we present key results produced from the model. Figures 1 and 2 show the probabilities of being in a specific regime for each process \( s_{c}^{v} \) and \( s_{c}^{f} \) over time. The probabilities are smoothed in the sense of Kim (1994); i.e., full sample information is used in getting the regime probabilities at each date.

When looking at the process in which equation coefficients are allowed to change (see \( s_{c}^{c} \) shown in Figure 1), it is apparent that Regime 1 (\( s_{c}^{c} = 1 \)) was prevailing during episodes of high inflationary pressure in the 1970s and 1980s, and dominant during the age of the 9/11 attacks, Dot-com bubble, and corporate scandals. This regime was also in place during the financial crisis originated by subprime mortgages, as well as during the European debt crisis. We thus label this regime as the distresse regime. All of the above-mentioned sub-periods, captured by this regime, contain the same similarities, namely major disruption in financial markets, macroeconomic imbalances, and heightened uncertainty. Regime 2 has prevailed for
the remaining years of the sample, characterized by episodes of tranquility. We label it as the *tranquil* regime. Regarding the process governing the structural disturbance variances, $s_t^v$, the model clearly captures three distinct regimes of volatility: a *low-, high-, and extreme-*volatility regime, as shown in Table 1.\footnote{Following Sims and Zha (2006), we normalize the size of shock variances to unity in Regime 1, $s_t^v = 1$.} Looking at Figure 2, the high-volatility regime (i.e., Regime 3) corresponds clearly to the *pre-Great Moderation* period, where the size of shock variances in output is relatively four times larger than those experienced in the low-volatility regime (i.e., Regime 1). The higher degree of volatility in the pre-1980s period corroborates, for example, with Kim and Nelson (1999). Finally, the extreme-volatility regime (i.e., Regime 2) identifies exceptional events, like the beginning of the Great Recession in 2008.

Table 1. Relative shock standard deviations across regimes.

<table>
<thead>
<tr>
<th></th>
<th>Production gdp</th>
<th>Uncertainty vix</th>
<th>Financial sp</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s_t = 1$</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
<tr>
<td></td>
<td>[1.0000;1.0000]</td>
<td>[1.0000;1.0000]</td>
<td>[1.0000;1.0000]</td>
</tr>
<tr>
<td>$s_t = 2$</td>
<td>4.5043</td>
<td>7.4577</td>
<td>9.1667</td>
</tr>
<tr>
<td></td>
<td>[2.0673;7.2627]</td>
<td>[3.2094;31.0974]</td>
<td>[5.3735;13.8848]</td>
</tr>
<tr>
<td>$s_t = 3$</td>
<td>4.6611</td>
<td>1.7173</td>
<td>0.9002</td>
</tr>
<tr>
<td></td>
<td>[3.4921;6.0990]</td>
<td>[1.0057;2.7058]</td>
<td>[0.5183;1.3942]</td>
</tr>
</tbody>
</table>
Tables 2 and 3 report estimated transition matrices at the posterior mode, with 68% probability intervals in brackets, for Markov-switching processes $Q^v$ and $Q^c$, respectively. Looking at the $s^c_t$ process, the distress regime ($q_{11}^c = 0.8969$) is slightly less persistent (an average duration of about 9 quarters) than the tranquil regime ($q_{22}^c = 0.9324$) which covers most of the sample with an average duration over 15 quarters. Looking at the $s^v_t$ process, Regimes 1 and 3 are unsurprisingly the most persistent, with $q_{11}^v = 0.9432$ and $q_{33}^v = 0.9846$, respectively. Regime 2, has a very short-lived duration of about 3 quarters. The tight interval probabilities reinforce the credibility of the estimated mode values.

In summary, our results suggest that the economy has experienced shocks whose the size change over time. Interestingly, the behavior of the economy — characterized by the systematic part of the model, i.e., equation coefficients — is different in distress periods than in tranquil periods. The objective of the next section is then to investigate the extent at which
III.3.2. Regime-dependent dynamic effects of uncertainty shocks. We illustrate possible differences in dynamics across the two regimes of the process governing equation coefficients, \( s_t^c \), by examining the response of the rest of the economy to a pure disturbance in uncertainty ("one-time uncertainty shock").\(^9\)

Figure 3 reports the impulse responses of endogenous variables across the two regimes. The first column shows the responses in the tranquil regime, while the responses in the stress regime are displayed in the second column. All of these panels display the deviation in percent for the series entered in log-levels (output), whereas it displays the deviation in percent points (p.p) for the VIX index and credit spread. The third column shows the differences between impulse responses of the two regimes. In any column, the dotted lines represent the median, with the 16th and 84th percentile displayed in solid lines. For comparability across regimes,

\(^9\)Here, we assume that a particular regime will last in the wake of the shock, although agents take into account the possibility of regime shifts. Alternatively, we could have employed the generalized impulse-response function (GIRF) developed by Koop, Pesaran, and Potter (1996), and transposed to MS-SVAR models by Karamé (2015). GIRF makes allowance for the dependence on initial conditions, future shocks, and future regimes.
Figure 3. Impulse-response functions to uncertainty shock under both regimes obtained from the identified MS-SVAR model. The first and second column report impulse responses of endogenous variables under distress and tranquil regimes, respectively. The last column displays the difference between the two regimes. In each case, the median is reported in dotted line and the 68% error bands in solid lines.

Our uncertainty shock is scaled to induce a 10 percentage points immediate increase in the VIX index.
Looking at this figure, the responses of our measure of aggregate activity do vary much over time, indicating that the differences among the two regimes in the coefficients of the system of equations are very large. After a positive innovation in our uncertainty measure that causes a 10 percentage points increase in the VIX index, the output falls slowly and moderately in the tranquil regime, but falls quickly and considerably in the distress regime, until reaching its minimum after 3 quarters. These differences seem to be statistically significant when taking into account the 68 percent probability intervals (right-top panel); error bands of the differences lie exclusively within the negative region over the first 8 quarters.

Interestingly, the response of credit spread is much larger in the distress regime, indicating credit costs for firms are relatively high. Once again, error bands reinforce these results. We thus might say that the amplification effects on output occur primarily through changes in credit spreads.

We investigate this intuition in the next section through inference of a MS-DSGE model by using the regime-dependent impulse responses obtained from the identified MS-SVAR model.

IV. A structural interpretation

This section provides a structural interpretation of the empirical results described in section III. Before discussing the estimation results in Section IV.3, we first present our micro-founded model in Section IV.1, as well as the solving method in Section IV.2. The full general equilibrium model is provided in Appendix C.

IV.1. A Markov-switching DSGE Model with financial frictions. In previous sections, we have shown that there are important differences in the transmission mechanism of uncertainty shocks between tranquil and distress regimes. In order to provide a structural interpretation of these changes, we need to develop a microfounded business cycle model whose key parameters are allowed to change across regimes. We, therefore, develop a DSGE model with a two-states Markov-switching process, $\chi_t$, that evolves according to the transition probabilities $p_{ij}$, with $i, j \in \{1, 2\}$.

The structure of the model is based on the Smets and Wouters (2007)’s model with Financial Frictions (hence after SWFF) developed by Del Negro, Giannoni, and Schorfheide (2015), which is a log-linearized version of the medium scale DSGE model with real and nominal frictions originally developed by Christiano, Eichenbaum, and Evans (2005), Smets
and Wouters (2007), and Christiano, Motto, and Rostagno (2014). In what follows, we briefly summarize the key ideas of the model.

Sticky nominal prices and wages adjust following a Calvo mechanism with probability $1 - \zeta_p$ and $1 - \zeta_w$, respectively, and with partial indexation $\iota_p$ and $\iota_w$, respectively. The nominal interest rate is set according to a Taylor rule, where the nominal interest rate responds to inflation ($\psi_1$), to output gap ($\psi_2$), and to its lagged value ($\rho R$). The model incorporates fixed cost ($\Phi_p$), a variable capital utilisation ($\psi$), and costs of adjusting the capital stock ($S''$) in the production sector. Households’ preferences are characterized by habit formation in consumption, governed by the parameter $h$.

The model includes also the Bernanke, Gertler, and Gilchrist (1999)’s financial accelerator mechanism, allowing us to introduce both financial frictions and uncertainty shocks into the model. Entrepreneurs receive funds from households’ deposits to banks and uses it, together with personal wealth, to purchase physical capital, which is rented to intermediate goods producers. Entrepreneurs experience idiosyncratic productivity shocks that affect their ability to manage capital. When their revenue is too low, they are not able to pay back bank loans. Banks protect themselves against default risk by charging a premium over the deposit rate. This premium varies exogenously due to changes in the dispersion of entrepreneurs’ idiosyncratic productivity, captured by $\sigma_{\omega,t}$, and endogenously as a function of the balance sheet of entrepreneurs, through the elasticity parameter $\zeta_{sp,b}$. These exogenous changes follow an AR(1) process, with the persistence parameter $\rho_{\sigma_\omega}$, and the shock variance $\sigma_{\sigma_\omega}$. We interpret these exogenous changes as the theoretical counterpart of the structural uncertainty shocks identified in the empirical MS-SVAR.

To maintain model tractability, we do not allow all structural parameters to change over time. We believe that there are three set of candidates for explaining the differences in economic dynamics between both regimes. The first is related to the capital expenditures, namely adjustment costs of investment, $S''$. The second comes from the financial frictions in the economy, through the elasticity of premium to net worth, $\zeta_{sp,b}$, and the persistence of uncertainty shocks, $\rho_{\sigma_\omega}$. The third is related to monetary policy rule, i.e., the response of nominal interest rate to inflation ($\psi_1$) and to output gap ($\psi_2$). The monetary authority might be tempted to change its behavior depending on the state of the economy. The remaining parameters remain invariant with time. The time-invariance shock variance, $\sigma_{\sigma_\omega}$, is justified
by our empirical results that show important changes in the transmission mechanism for a
given constant shock variance.

IV.2. **Solving MS-DSGE model.** We proceed in several steps to implement our regime-
switching model following Bianchi (2013) and Lhuissier and Zabelina (2015). First, because
the economy exhibits a trend, we stationarize variables by their corresponding trend. Second,
we compute the steady state of the stationary model and then we log-linearize it around its
steady state. Third, we add the index $\chi_t$, the exogenous first-order Markov process, to the
model. The compact form of the model becomes as follows:

$$A(\chi_t)f_t = B(\chi_t)f_{t-1} + \Psi(\chi_t)\varepsilon_t + \Pi(\chi_t)\eta_t,$$

where $f_t$ is the vector of endogenous components, $\varepsilon_t$ is a vector of exogenous shocks, and
$\eta_t$ is a vector of expectational errors.

We employ the solution algorithm based on the Mean Square Stable (MSS) concept pro-
posed by Farmer, Waggoner, and Zha (2009), Farmer, Waggoner, and Zha (2011), and Cho
(2016). Such algorithms allow agents to take into account the possibility of future regime
shifts when forming expectations. For efficiency and speed reasons, we use the Cho (2016)'s
algorithm, which uses a forward method.

IV.3. **Empirical Results.** This section provides the main quantitative results from the
estimated MS-DSGE model. First, we present our estimation strategy in Section IV.3.1. Second,
we report the estimates of structural parameters in Section IV.3.2. Third, we present,
in Section IV.3.3, the impulse response functions to uncertainty shock.

IV.3.1. **Estimation strategy.** Our estimation strategy is analogous to the impulse response
matching approach used by Rotemberg and Woodford (1997) and Christiano, Eichenbaum,
and Evans (2005), except that we are estimating the parameters to fit our regime-dependent
impulse responses from a MS-SVAR, as opposed to impulse responses from a constant-
parameters SVAR.\textsuperscript{10} Our empirical analysis matches the estimated impulse responses functions of output and credit spread, but we do not include the VIX index, which is not observable in the theoretical model. To the best of our knowledge, Basu and Bundick (2017)

\textsuperscript{10}We thank Mathias Trabandt for sharing computer codes used in Christiano, Trabandt, and Walentin (2010) on inference of constant DSGE models with the standard impulse response matching approach. We adapt their codes into Markov-switching environment.
are the first to define the VIX index in a DSGE model, but this requires a third-order approximation to the model policy functions. At this stage, there is no efficient estimation algorithm to allow high-order approximations for MS-DSGE models. Nevertheless, it should be stressed that Foerster, Rubio-Ramírez, Waggoner, and Zha (2016) attempt to fill part of this gap using perturbation methods. However, their solution methods is not enough fast and accurate to be used in an estimation algorithm.

Let $\xi$ is a $N \times 1$ vector, which stack the contemporaneous and 15 lagged responses to each of two endogenous variables to the uncertainty shock. The number of elements in $\xi$ is equal to $2$ (i.e., the number of regimes) times $2$ (i.e., the number of variables) times $16$ (i.e., the horizon) = 64 elements. Let $\xi(\theta)$ denotes the mapping from $\theta$ to the MS-DSGE model impulse response functions, with $\theta$ is a vector containing all estimated parameters. The likelihood function of the data, $\tilde{\xi}$ is defined as as function of $\theta$:

$$f(\tilde{\xi}|\theta, \bar{V}) = \left(\frac{1}{2\pi}\right)^{\frac{N}{2}} |\bar{V}^{-\frac{1}{2}}| \times \left[-\frac{1}{2}(\tilde{\xi} - \xi(\theta))'\bar{V}^{-1}(\tilde{\xi} - \xi(\theta))\right],$$

where $\bar{V}$ is a diagonal matrix with the sample variances of the $\tilde{\xi}$'s along the diagonal. Conditional on $\tilde{\xi}$ and $\bar{V}$, the Bayesian posterior of $\theta$ is as follows:

$$f(\theta, \bar{V}) \propto f(\tilde{\xi}|\theta, \bar{V}) \times f(\theta),$$

where $f(\theta)$ denotes the priors on $\theta$.

The strategy of estimation begins by maximizing (6) using the CSMINWEL program, the optimization routine developed by Christopher A. Sims. Once at the posterior mode, we can start a Markov Chain Monte Carlo method to sample the posterior distribution. More specifically, we employ the Random-walk Metropolis Hasting procedure to generate draws from the joint posterior distribution of the MS-DSGE model. The results shown in the paper is based on 50,000 draws. We discard the first 10 percent draws as burn-in, and every 10th draws is retained.

IV.3.2. Estimates of key parameters. In order to keep the estimation procedure tractable, we calibrate several parameters. Most of them are set along the line of those estimated by Del Negro, Giannoni, and Schorfheide (2015). Table 4 summarizes it. Note also that the transition probabilities ($p_{ij}$) are calibrated so that to be equal to those from the transition
matrix $Q_c$, which governs regime changes in the equation coefficients (and thus in transmission mechanism) of the estimated MS-SVAR model in Section III.

Table 4. Calibration of structural parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>Capital share</td>
<td>0.1687</td>
</tr>
<tr>
<td>$\zeta_p$</td>
<td>Calvo prices</td>
<td>0.7467</td>
</tr>
<tr>
<td>$\nu_p$</td>
<td>Price indexation</td>
<td>0.2684</td>
</tr>
<tr>
<td>$\Upsilon$</td>
<td>Technological progress</td>
<td>1.0000</td>
</tr>
<tr>
<td>$h$</td>
<td>Consumption habit</td>
<td>0.4656</td>
</tr>
<tr>
<td>$\nu_t$</td>
<td>Elasticity labor</td>
<td>0.1047</td>
</tr>
<tr>
<td>$\zeta_w$</td>
<td>Calvo wages</td>
<td>0.7922</td>
</tr>
<tr>
<td>$\nu_w$</td>
<td>Wage indexation</td>
<td>0.5729</td>
</tr>
<tr>
<td>$\Phi_p$</td>
<td>Fixed costs</td>
<td>4.5260</td>
</tr>
<tr>
<td>$\psi$</td>
<td>Elastic. capital utilization costs</td>
<td>0.1800</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^*$</td>
<td>SS quarterly inflation</td>
<td>0.5465</td>
</tr>
<tr>
<td>$\sigma_c$</td>
<td>Elasticity utility</td>
<td>1.5073</td>
</tr>
<tr>
<td>$\rho_R$</td>
<td>Taylor rule smoothing</td>
<td>0.8519</td>
</tr>
<tr>
<td>$F(\omega)$</td>
<td>Default rate</td>
<td>0.0300</td>
</tr>
<tr>
<td>$sp^*$</td>
<td>SS quarterly spread</td>
<td>1.1791</td>
</tr>
<tr>
<td>$\gamma^*$</td>
<td>Survival rate</td>
<td>0.9900</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>SS quarterly growth rate</td>
<td>0.4010</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Discount factor</td>
<td>0.7420</td>
</tr>
<tr>
<td>$p_{11}$</td>
<td>Prob. staying in Regime 1</td>
<td>0.8969</td>
</tr>
<tr>
<td>$p_{22}$</td>
<td>Prob. staying in Regime 2</td>
<td>0.9324</td>
</tr>
</tbody>
</table>

Note: Calibration is based on the estimated parameters in Del Negro, Giannoni, and Schorfheide (2015), except for transition matrix parameters, which are those obtained, at the mode, from the identified MS-SVAR model.

Table 5 reports the specific distribution, the mean and the standard deviation for each estimated parameter. Most of the prior distributions for the parameters follow those in Del Negro, Giannoni, and Schorfheide (2015). The prior for costs of investment adjustment follows a gamma distribution with the mean 1.00 and the standard deviation 0.75. Regarding monetary policy parameters, the prior for the responses to inflation follows a normal distribution with the mean 1.00 and the standard deviation 0.20, and the prior for the responses to output gap has a gamma distribution with the mean 0.12 and the standard deviation 0.10. The prior for the parameters of financial contract is rather dispersed and cover a large parameter space. We employ a uniform distribution defined over $[0; 0.10]$. The prior distributions of uncertainty shock process is weakly informative. We use a beta distribution for the persistence of the shock with the mean 0.75 and standard deviation 0.15. Regarding the shock variance, we impose an inverted gamma distribution, with hyper-parameters, $\nu$ and $s$, equal to 0.05 and 20.00, respectively.11

11The inverted gamma distribution is as follows $p_{IG}(\sigma | \nu, s) \propto \sigma^{-\nu-1}e^{-\nu \sigma^2/2\sigma^2}$. 
Table 5. Prior and posterior distribution.

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Description</th>
<th>Prior</th>
<th>Posterior</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Density</td>
<td>para(1)</td>
<td>para(2)</td>
<td>Mode</td>
<td>[5; 95]</td>
<td>para(1)</td>
<td>para(2)</td>
</tr>
<tr>
<td>$S''(k = 1)$</td>
<td>Investment adjustment costs</td>
<td>G 0.75</td>
<td>0.50</td>
<td>0.2439</td>
<td>0.0751</td>
<td>0.5145</td>
<td></td>
</tr>
<tr>
<td>$S''(k = 2)$</td>
<td>Investment adjustment costs</td>
<td>G 0.75</td>
<td>0.50</td>
<td>0.7638</td>
<td>0.1831</td>
<td>1.6440</td>
<td></td>
</tr>
<tr>
<td>$\phi_1(k = 1)$</td>
<td>Taylor rule, inflation</td>
<td>N 1.70</td>
<td>0.20</td>
<td>1.7099</td>
<td>1.3775</td>
<td>2.0690</td>
<td></td>
</tr>
<tr>
<td>$\phi_1(k = 2)$</td>
<td>Taylor rule, inflation</td>
<td>N 1.70</td>
<td>0.20</td>
<td>1.6960</td>
<td>1.3751</td>
<td>2.0394</td>
<td></td>
</tr>
<tr>
<td>$\phi_2(k = 1)$</td>
<td>Taylor rule, output gap</td>
<td>G 0.12</td>
<td>0.10</td>
<td>0.0203</td>
<td>0.0027</td>
<td>0.0459</td>
<td></td>
</tr>
<tr>
<td>$\phi_2(k = 2)$</td>
<td>Taylor rule, output gap</td>
<td>G 0.12</td>
<td>0.10</td>
<td>0.0705</td>
<td>0.0130</td>
<td>0.1709</td>
<td></td>
</tr>
<tr>
<td>$\zeta_{sp,b}(k = 1)$</td>
<td>Elas. financial contract</td>
<td>U 0.00</td>
<td>0.10</td>
<td>0.0501</td>
<td>0.0238</td>
<td>0.0896</td>
<td></td>
</tr>
<tr>
<td>$\zeta_{sp,b}(k = 2)$</td>
<td>Elas. financial contract</td>
<td>U 0.00</td>
<td>0.10</td>
<td>0.0027</td>
<td>0.0008</td>
<td>0.0057</td>
<td></td>
</tr>
<tr>
<td>$\rho_{\sigma_{\omega}}(k = 1)$</td>
<td>Persistence uncertainty shock</td>
<td>B 0.75</td>
<td>0.15</td>
<td>0.9270</td>
<td>0.8665</td>
<td>0.9763</td>
<td></td>
</tr>
<tr>
<td>$\rho_{\sigma_{\omega}}(k = 2)$</td>
<td>Persistence uncertainty shock</td>
<td>B 0.75</td>
<td>0.15</td>
<td>0.7957</td>
<td>0.7331</td>
<td>0.8477</td>
<td></td>
</tr>
<tr>
<td>$\sigma_{\sigma_{\omega}}$</td>
<td>Uncertainty shock</td>
<td>Inv-G 0.04</td>
<td>20.0</td>
<td>0.0399</td>
<td>0.0327</td>
<td>0.0496</td>
<td></td>
</tr>
</tbody>
</table>

Note: N stands for Normal, B Beta, G for Gamma, Inv-G for Inverted-Gamma and U for Uniform distributions. The 5 percent and 95 percent demarcate the bounds of the 90 percent probability interval. Para(1) and Para(2) correspond to the means and standard deviations for the normal, beta and gamma distributions, to $\nu$ and $s$ for the inverted-gamma distribution, where $p_{IG}(\sigma|\nu,s) \propto \sigma^{-\nu-1}e^{-\nu s^2/2\sigma^2}$, and to the lower and upper bound for the uniform distribution.

The group of estimated parameters is stacked as follows:

$$\theta = [S''(k), \psi_1(k), \psi_2(k), \zeta_{sp,b}(k), \rho_{\sigma_{\omega}}(k), \sigma_{\sigma_{\omega}}], \quad \text{with} \quad k = \{1, 2\}. \quad (7)$$

The last three columns of Table 5 report the posterior mode with the 90 percent probability interval for each structural parameter. Clearly, none of capital and monetary policy parameters seems to account for the differences in the dynamics between the two regimes. The estimates for $\psi_1(k)$, under both regimes, indicate that the posterior mode is closely similar to the mean of the prior, meaning our impulse responses contain little information about the response of monetary authority to inflation. The estimates for $\psi_2(k)$, the monetary response to output gap, are relatively small for each regime, where $\psi_2(k = 1) = 0.02$ and $\psi_2(k = 2) = 0.07$ at the mode. Note also that their 90 percent probability intervals overlap, meaning that there are no significant differences between those parameters across the two regimes. The estimates for $S''(k)$ are about 0.24 and 0.76 in Regimes 1 and 2, respectively,
slightly lower than those reported in the literature. Once again, the 90 percent probability intervals overlap, meaning that their differences do not appear to be fairly significant.

By contrast, the parameter of the financial contract, $\zeta_{sp,b}(k)$, differs considerably between the two regimes. At the posterior mode, its estimate is close to zero in the tranquil regime, but turns out to be relatively high in the distress regime, for a value of 0.0501. The fact that the probability intervals do not overlap reinforces our results. It follows that, in distress periods, linkages between the quality of borrowers balance sheets and their access to external finance are strengthened, implying thus a more powerful financial accelerator mechanism. More details about this mechanism is provided in the next section. Clearly, this finding shows that the macroeconomic impact of uncertainty shocks depend on the degree of financial frictions in the economy.

We can easily recover the values of monitoring costs, the deep parameter of the financial accelerator, from $\zeta_{sp,b}(k)$, the sensitivity of the external finance premium to leverage ratio. Under Regime 1 (i.e., distress regime), lenders pay monitoring costs which account for about 9 percent (at the mode) of the realized gross payoff to the firm’s capital to observe an individual borrower’s realized return. These costs are much higher than those observed in Regime 2 (i.e., tranquil regime), i.e., about 1 percent (at the mode) of the realized gross payoff to the firm’s capital.

Recently, Fuentes-Albero (2018) emphasizes the crucial role played by time-varying monitoring costs in shaping the business cycles. Our approach is, however, substantially different. Fuentes-Albero (2018) considers, in some way, shocks to the monitoring cost which generates the impulsion at the origin of business cycles, while in our approach, changes in the monitoring cost represent the amplification and propagation mechanisms of uncertainty shocks. In this respect, the Lindé, Smets, and Wouters (2016) specification is closest to our approach. They estimate, with full information methods, a DSGE model with financial frictions à la Bernanke, Gertler, and Gilchrist (1999) in which the monitoring cost is allowed to change according to a Markov-switching process. Interestingly, they capture changes in the degree of the financial frictions, with repeated changes in the monitoring costs between a low (2.90 percent) and high (8.40 percent) value over time. These estimated values corroborate with our finding, except that the times of monitoring cost changes are slightly different. These differences can be explained by two main reasons. First, they estimate a MS-DSGE with
full information methods — i.e., key macroeconomic and financial variables are directly observable in the model — while we estimate our MS-DSGE by the impulse-response matching approach. Second, our MS-SVAR model takes properly into account heteroskedasticity of U.S. macroeconomic disturbances, while they do not. Indeed, Sims (2001), and more recently Lhuissier and Zabelina (2015), have shown the importance of capturing heteroskedasticity before allowing changes in economic dynamics in order to avoid misleading results. In Lindé, Smets, and Wouters (2016), only the monitoring cost parameter is allowed to change over time while shock variances remain constant. Our paper overcomes this issue by allowing both equation coefficients and shock variances to change over time independently.\footnote{Interestingly, in Lindé, Smets, and Wouters (2016), the times of changes in the monitoring cost parameter coincide remarkably well with the times of changes of the high-volatility regime reported in Figure 2, suggesting a biased estimation of their regime-dependent monitoring cost.}

IV.3.3. Impulse responses. Figure 4 reports, in red line, the impulse responses of endogenous variables to the uncertainty shock obtained from the MS-DSGE model. The first column represents the responses under the distress regime, while the second column represents those in the tranquil regime. For comparison purposes, we also present the 68 percent probability intervals of the MS-SVAR model-implied responses. A number of results are worth emphasizing here. First, the model performs well at accounting for the dynamic responses of the economy to a uncertainty shock. All the DSGE model-implied responses lie within the 68 percent probability intervals computed from the MS-SVAR model. From a qualitative point of view, the responses of the output and the credit spread in the tranquil regime share some common features with the responses in the distress regime. Credit spread and output move in opposite directions; output declines progressively, while credit spread rises immediately and then begins to return its pre-shock level steadily.

The transmission mechanism is straightforward. The uncertainty shock directly alters the degree of risk associated with the asymmetric information between lenders and entrepreneurs who borrow external funds to produce physical capital goods. It moves the dispersion of entrepreneurs’ idiosyncratic productivity. With imperfect financial markets, this shock implies higher agency costs since more entrepreneurs draw low levels of productivity and are then unable to reimburse their debts. Then, a positive uncertainty shock increases both the risk of default and the cost of external funds which lead to a fall in the economic activity of
entrepreneurs transmitted to the overall economy in general equilibrium through an increase of the credit spread and a fall in investment and production. Say it differently, financial frictions act as the main mechanism through which changes in uncertainty affect macroeconomic variables.

Furthermore, the model succeeds in accounting for the differences in the responses of endogenous variables across the two regimes. Indeed, there is a notable change in the way

**Figure 4.** Impulse-response functions to a uncertainty shock. For each regime (i.e., each column), the median responses from the identified MS-BVAR model is reported in dotted blue line and the 68% error bands in solid blue lines. The red line reports the responses (at the mode) from the MS-DSGE model.
both output and credit spread respond to the shock. Concerning the changes in the impulse responses between the two regimes, the responses under the distress regime are remarkably amplified compared to those in the tranquil regime. Under these circumstances, financial frictions act as an amplification mechanism. Note also that we observe a stronger recovery in real activity in bad times.

This stronger effect of uncertainty in distress periods can be explained as follows. The elasticity parameter of financial contract, $\zeta_{sp,b}$, relates our measure of the external finance premium (i.e., credit spread) to the firm’s net worth. Under high stress, $\zeta_{sp,b}(k = 1) = 0.0501$, the premium becomes much more sensitive to a firm’s net worth, compared to tranquil periods ($\zeta_{sp,b}(k = 2) = 0.0027$). In this context, a uncertainty shock causes larger credit spread increases, and therefore, larger and long-lasting negative effects in economic activity. In contrast, when stress is low, the economy is better capable of absorbing the coming economic shocks. As a result, the macroeconomic effects are less pronounced.

IV.4. **Expectation effects of regime shifts in financial conditions.** In the previous section, we have illustrated the role of financial frictions in propagating uncertainty shocks by comparing economic outcomes of two possible regimes: one regime with a high elasticity of the credit spread to the net worth position, and another regime with a low-degree of financial frictions, i.e., a low elasticity in financial contract. Results were not only based on the estimated structural parameters of each regime, but also on the transition matrix used by agents when forming their expectations. In this section, we gauge what would have happened if agents had considered different probabilities of moving across regimes. Such a counterfactual is interesting to execute because it allows assessing the role of expectation effects of regime switching in financial conditions.

Figure 5 displays the impulse responses of variables following a uncertainty shock when the probability of staying in the same regime varies between 0.00 to 1.00. Each column represents the response of variables under a specific regime. When considering $p_{ii} = 1$, agents believe that the regime in which they are will last indefinitely. Inversely, the more $p_{ii}$ declines, the more agents believe that the economy will move to the other regime in the next period.

Clearly, the expectation effects play an important role in shaping the dynamic behavior of macroeconomic variables. As one can see, if agents take into account the effects of possible changes in future financial conditions, macroeconomic outcomes are remarkably altered. The
more are agents optimistic about future financial conditions (i.e., gradual moves toward $p_{ii} = 0$ in the left column (Regime 1) or $p_{ii} = 1$ in the right column (Regime 2)), the more macroeconomic effects are dampened. Reciprocally, pessimism of agents about financial conditions (i.e., gradual moves toward $p_{ii} = 1$ in the right column or $p_{ii} = 0$ in the left column) amplify the effects of uncertainty shocks. Quantitatively, the expectation effects appear to be bigger under the distress regime, where the output effects of the shock can be divided by three. The role of expectation effects of regime switching in the degree of financial frictions appears to be important in amplifying or mitigating the propagation of uncertainty.
shocks. Therefore, these expectation effects are an important component of the financial accelerator mechanism.

V. Conclusion

Why are the real effects of uncertainty shocks so different over time? Our results point to a key role for changes in the degree of financial frictions; the financial accelerator is strengthened in distress periods. Under these circumstances, agents’ expectations around the level of frictions can alter macroeconomic outcomes. Optimistic expectations about future financial conditions dampen contractionary effects of uncertainty shocks on aggregate activity. Conversely, pessimistic expectations amplify their effects.

These conclusions have important implications for the conduct of monetary and macro-prudential policies. For example, the bulk of the evidence suggests that these policies can reduce the frequency and severity of financial disruptions, and thus the likelihood of observing a regime characterized by a high degree of financial frictions. In this context, if policymakers communicate to and persuade, in a clear way, agents that such policies are around the corner, then they can, even before implementing them, dampen the adverse effects of uncertainty shocks. The ability of policymakers to manage agents’ expectations reveals to be crucial in shaping business cycle fluctuations.
Appendix A. Data

All data are organized quarterly from the second Quarter of 1962 to the second Quarter of 2018. Most data comes from Federal Reserve Economic Database (FRED).

- $gdp_t$: output is the real GDP (GDPC1).
- $vix_t$: uncertainty is the Chicago Board of Options Exchange Market Volatility Index. From 1963 to 2009, we use the constructed index by Bloom (2009). Then, from 2009, we follow Stock and Watson (2012) and take a quarterly average of daily VIX.
- $sp_t$: credit spread is constructed as the difference between BAA corporate bond yields (BAA) and AAA corporate bond yields (AAA).

For inference, we use the natural log of output. Our spread and uncertainty variables remain unchanged.

Appendix B. Markov-switching Structural Bayesian VAR model

This section provides a detailed description of the Bayesian inference employed in this paper. More specifically, we closely follow Sims, Waggoner, and Zha (2008).

B.1. The posterior. Before describing the posterior distribution, we introduce the following notation: $\theta$ and $q$ are vectors of parameters where $\theta$ contains all the parameters of the model (except those of the transition matrix) and $q = (q_{i,j}) \in \mathbb{R}^{h^2}$. $Y_t = (y_1, \ldots, y_t) \in (\mathbb{R}^n)^t$ are observed data with $n$ denoting the number of endogenous variables and $S_t = (s_0, \ldots, s_t) \in H^{t+1}$ with $H \in \{1, \ldots, h\}$.

The log-likelihood function, $p(Y_T|\theta, q)$, is combined with the prior density functions, $p(\theta, q)$, to obtain the posterior density, $p(\theta, q|Y_T) = p(\theta, q)p(Y_T|\theta, q)$.

B.1.1. The likelihood. Following Hamilton (1989), Sims and Zha (2006), and Sims, Waggoner, and Zha (2008), we employ a class of Markov-switching structural VAR models of the following form:

$$y_t' A(s_t) = x_t' F(s_t) + \varepsilon_t' \Xi^{-1}(s_t),$$

with $x_t' = \begin{bmatrix} y_{t-1}' & \cdots & y_{t-\rho}' & 1 \end{bmatrix}$ and $F(s_t) = \begin{bmatrix} A_1(s_t) & \cdots & A_{\rho}(s_t) & C(s_t) \end{bmatrix}'$. 


Let \( a_j(k) \) be the \( j \)th column of \( A(k) \), \( f_j(k) \) be the \( j \)th column of \( F(k) \), and \( \xi_j(k) \) be the \( j \)th diagonal element of \( \Xi(k) \). The conditional likelihood function is as follows:

\[
p(y_t|s_t, Y_{t-1}) = |A(s_t)| \prod_{j=1}^{n} |\xi_j(s_t)| \exp \left( -\frac{\xi_j^2(s_t)}{2} (y'_t a_j(s_t) - x'_t f_j(s_t))^2 \right).
\]

(9)

To simplify the Gibbs-sampling procedure described in the next section, it is preferable to rewrite the condition likelihood function with respect to free parameters from matrix \( A(s_t) \) and \( F(s_t) \):

\[
|A(s_t)| \prod_{j=1}^{n} |\xi_j(s_t)| \exp \left( -\frac{\xi_j^2(s_t)}{2} (y'_t + x'_t W_j b_j(s_t) - x'_t V_j g_j(s_t))^2 \right),
\]

(10)

where \( a_j(s_t) = U_j b_j(k) \) and \( f_j(s_t) = V_j g_j - W_j U_j b_j(k) \) is a result from the linear restrictions

\[
R_j \begin{bmatrix} a_j & f_j \end{bmatrix} = 0;
\]

and \( U_j \) and \( V_j \) are matrices with orthonormal columns and \( W_j \) is a matrix. See Waggoner and Zha (2003) for further details.

The log likelihood function is given by

\[
p(Y_T|\theta, q) = \sum_{t=1}^{T} \ln \left\{ \sum_{s_t=1}^{h} p(y_t|s_t, Y_{t-1}) \Pr [s_t|Y_{t-1}] \right\},
\]

(11)

where

\[
\Pr [s_t = i|Y_{t-1}] = \sum_{j=1}^{h} \Pr [s_t = i, s_{t-1} = j|Y_{t-1}]
\]

\[
= \sum_{j=1}^{h} \Pr [s_t = i|s_{t-1} = j] \Pr [s_{t-1} = j|Y_{t-1}].
\]

(12)

(13)

with \( q_{i,j} = \Pr [s_t = i|s_{t-1} = j] \) are the transition probabilities from the \( h \times h \) matrix \( Q \)

\[
Q = \begin{bmatrix}
q_{1,1} & \cdots & q_{1,j} \\
\vdots & \ddots & \vdots \\
q_{i,1} & \cdots & q_{i,j}
\end{bmatrix}
\]

(14)

The probability terms are updated as follows:

\[
\Pr [s_t = j|Y_t] = \Pr [s_t = j|Y_{t-1}, y_t] = \frac{p(s_t = j, y_t|Y_{t-1})}{p(y_t|Y_{t-1})}
\]

\[
= \frac{p(y_t|s_t = j, Y_{t-1}) \Pr [s_t = j|Y_{t-1}]}{\sum_{j=1}^{h} p(y_t|s_t = j, Y_{t-1}) \Pr [s_t = j|Y_{t-1}]}.
\]

(15)

(16)
B.1.2. The prior. Following Sims and Zha (1998), we exploit the idea of a Litterman’s random-walk prior from structural-form parameters. Note that dummy observations are not introduced as a component of the prior to keep in line with the original Litterman’s prior. Using linear restrictions, the overall prior, \( p(\theta, q) \), is given in the following way:

\[
p(b_j(k)) = \text{normal}(b_j(k)|0, \overline{\Sigma}_{b_j}), \tag{17}
\]

\[
p(g_j(k)) = \text{normal}(g_j(k)|0, \overline{\Sigma}_{g_j}), \tag{18}
\]

\[p(\xi_j^2(k)) = \text{gamma}(\xi_j^2(k)|\bar{\alpha}_j, \bar{\beta}_j), \tag{19}\]

\[p(q_j) = \text{dirichlet}(q_{i,j}|\alpha_{1,j}, \ldots, \alpha_{k,j}), \tag{20}\]

where \( \overline{\Sigma}_{b_j}, \overline{\Sigma}_{g_j}, \) and \( \overline{\Sigma}_{\delta_j} \) denotes the prior covariance matrices and \( \bar{\alpha}_j \) and \( \bar{\beta}_j \) are set to one, allowing the standard deviations of shocks to have large values for some regimes.

The Gamma distribution is defined as follows:

\[
gamma(x|\alpha, \beta) = \frac{1}{\Gamma(\alpha)} \beta^{\alpha} x^{\alpha - 1} e^{-\beta x}. \tag{21}\]

Regarding the transition matrix, \( Q \), suppose that \( q_j = [q_{1,j}, \ldots, q_{h,j}]' \). The prior, denoted \( p(q_j) \), follows a Dirichlet form as follows:

\[
p(q_j) = \left( \frac{\Gamma \left( \sum_{i \in H} \alpha_{i,j} \right)}{\prod_{i \in H} \Gamma(\alpha_{i,j})} \right) \times \prod_{i \in H} (q_{i,j})^{\alpha_{i,j} - 1}, \tag{22}\]

where \( \Gamma \) denotes the standard gamma function.

B.2. Gibbs-sampling. Following Kim and Nelson (1999) and Sims, Waggoner, and Zha (2008), a Markov Chain Monte Carlo (MCMC) simulation method is employed to approximate the joint posterior density, \( p(\theta, q, S_T|Y_T) \). The advantage of using VARs is that conditional distributions like \( p(S_T|Y_T, \theta, q) \), \( p(q|Y_T, S_T, \theta) \), and \( p(\theta|Y_T, q, S_T) \) can be obtained in order to exploit the idea of Gibbs-sampling by sampling alternatively from these conditional posterior distributions.
B.2.1. Conditional posterior densities, \( p(\theta|Y_T, q, S_T) \). To simulate draws of \( \theta \in \{b_j(k), g_j(k), \xi_j^2\} \) from \( p(\theta|Y_T, S_t, q) \), one can start to sample from the conditional posterior

\[
p(b_j(k)|y_t, S_t, b_i(k)) = \exp \left( -\frac{1}{2} b_j'(k) \Sigma_{b_j}^{-1} b_j(k) \right) \times \prod_{t \in \{t : s_t = k\}} \left[ |A(k)| \exp \left( -\frac{\xi_j^2(s_t)}{2} (y'_t a_j(k) - x'_t f_j(k))^2 \right) \right], \tag{23}
\]

using the Metropolis-Hastings (MH) algorithm. Then a multivariate normal distribution is employed to draw \( g_j(k) \):

\[
p(g_j(k)|y_t, S_t) = \text{normal}(g_j(k)|\tilde{\mu}_{g_j}(k), \tilde{\Sigma}_{g_j}(k)). \tag{24}
\]

The computational details of the posterior mean vectors and covariance matrices are given in Sims, Waggoner, and Zha (2008).

Disturbance variances \( \xi_j^2 \) are simulated from a gamma distribution

\[
p(\xi_j^2(k)|y_t, S_t) = \text{gamma}(\xi_j^2(k)|\tilde{\alpha}_j(k), \tilde{\beta}_j(k)), \tag{25}
\]

where \( \tilde{\alpha}_j(k) = \bar{\alpha}_j + \frac{T_{2,k}}{2} \) and

\[
\tilde{\beta}_j(k) = \bar{\beta}_j + \frac{1}{2} \sum_{t \in \{t : s_{2t} = k\}} (y'_t a_j(s_t) - x'_t f_j(s_t))^2, \tag{26}
\]

with \( T_{2,k} \) denoting the number of elements in \( \{t : s_{2t} = k\} \).

B.2.2. Conditional posterior densities, \( p(S_T|Y_T, \theta, q) \). A multi-move Gibbs-sampling is employed to simulate \( S_t, t = 1, 2, ..., T \). First, draw \( s_t \) according to

\[
p(s_t|y_t, S_t) = \sum_{s_{t+1} \in H} p(s_t|Y_T, \theta, q, s_{t+1})p(s_{t+1}|Y_T, \theta, q), \tag{27}
\]

where

\[
p(s_t|Y_T, \theta, q, s_{t+1}) = \frac{q_{s_{t+1},s_t}p(s_t|Y_T, \theta, q)}{p(s_{t+1}|Y_T, \theta, q)}. \tag{28}
\]

Then, in order to generate \( s_t \), one can use a uniform distribution between 0 and 1. If the generated number is less than or equal to the calculated value of \( p(s_t|y_t, S_t) \), we set \( s_t = 1 \). Otherwise, \( s_t \) is set equal to 0.
B.2.3. Conditional posterior densities, \( p(q|Y_T, S_T, \theta) \). The conditional posterior distribution of \( q_j \) is as follows:

\[
p(q_j|Y_t, S_t) = \prod_{i=1}^{h} (q_{i,j})^{n_{i,j} + \beta_{i,j} - 1},
\]

where \( n_{i,j} \) is the number of transitions from \( s_{t-1} = j \) to \( s_t = i \).

**Appendix C. The Markov-switching DSGE Model**

This section presents the Markov-switching structure of the DSGE model with financial frictions, originally developed by Del Negro, Giannoni, and Schorfheide (2015).

The model describes the dynamics of the following set of variables: \( c_t \) which stands for consumption, \( l_t \) for labor supply, \( R_t \) for the nominal interest rate, \( \pi_t \) for inflation, \( i_t \) for the level of investment, \( q^k_t \) for the value of capital in terms of consumption, \( r^k_t \) is the rental rate of capital, \( u_t \) for the utilization rate of physical capital, \( \bar{k} \) for the physical capital stock, \( k_t \) for the amount of physical capital effectively rented out to firms, \( w^h_t \) for the household’s marginal rate of substitution between consumption and labor, \( y_t \) for the output, and \( y^f_t \) for the output in the flexible price/wage economy.

The log-linearized equilibrium conditions are given for the stationary variables and the symbol \( * \) denotes the steady state value of the variable. The structural parameters of the economy impact the equilibrium conditions for the level of consumption,

\[
c_t = \frac{(1 - he^{-\gamma})}{\sigma_c (1 + he^{-\gamma})} (R_t - E_t [\pi_{t+1}]) + \frac{he^{-\gamma}}{(1 + he^{-\gamma})} c_{t-1} + \frac{1}{(1 + he^{-\gamma})} E_t [c_{t+1}] + \frac{(\sigma_c - 1)}{\sigma_c (1 + he^{-\gamma})} \frac{w^h_t l^*_t}{c^*_t} (l_t - E_t [l_{t+1}]),
\]

for the labor input,

\[
w^h_t = \frac{1}{1 - he^{-\gamma}} (c_t - he^{-\gamma} c_{t-1}) + \nu l_t,
\]

for the level of investment,

\[
q^k_t = S''(\chi_t)e^{2\gamma} (1 + \bar{\beta}) \left( i_t - \frac{1}{1 + \beta} t_{t-1} - \frac{\bar{\beta}}{1 + \beta} E_t [i_{t+1}] \right),
\]

for the utilization rate of physical capital,

\[
\frac{1 - \psi}{\psi} r^k_t = u_t,
\]
given the production technology of the final good,

\[ y_t = \Phi_p (\alpha k_t + (1 - \alpha) l_t), \tag{34} \]

the law of motion of the physical capital stock \( \bar{k}_t \),

\[ \bar{k}_t = \left(1 - \frac{i_s}{k_s}\right) \bar{k}_{t-1} + \frac{i_s}{k_s} i_t, \tag{35} \]

where \( i_s/k_s \) is the steady-state ratio of investment to capital, and the expression for the physical capital effectively used in production

\[ k_t = u_t + \bar{k}_t. \tag{36} \]

In these equations, the parameter \( \sigma_c \) captures the degree of relative risk aversion, \( h \) the degree of habit persistence in the utility function, \( S''(\chi_t) \) the second derivative of the adjustment cost function, \( \delta \) for the depreciation rate, \( \beta = \beta e^{(1 - \sigma_c)\gamma} \) the intertemporal discount rate, \( \sigma_c \) the degree of relative risk aversion, \( \psi \) the costs of capital utilization, \( \Phi_p \) the fixed cost of production, \( \alpha \) the income share of physical capital in the production function, \( \nu_l \) the curvature of the disutility of labor, and \( \gamma \) the steady-state growth rate.

The Phillips curves for prices \( (\pi_t) \) and wages \( (w_t) \) are, respectively,

\[ \pi_t = \frac{(1 - \zeta_p\beta) (1 - \zeta_p)}{(1 + \nu_p\beta) \zeta_p ((\Phi_p - 1) \epsilon_p + 1)} (w_t + \alpha l_t - \alpha k_t) \]

\[ + \frac{\nu_p}{1 + \nu_p\beta} \pi_{t-1} + \frac{\beta}{1 + \nu_p\beta} E_t [\pi_{t+1}], \tag{37} \]

and

\[ w_t = \frac{(1 - \zeta_w\beta) (1 - \zeta_w)}{(1 + \beta) \zeta_w ((\lambda_w - 1) \epsilon_w + 1)} (w_t^h - w_t) - \frac{1 + \nu_w\beta}{1 + \beta} \pi_t \]

\[ + \frac{1}{1 + \beta} \left( \nu_w \pi_{t-1} + \frac{\beta}{1 + \beta} E_t [w_{t+1} + \pi_{t+1}] \right), \tag{38} \]

where the parameters \( \zeta_p, \nu_p, \epsilon_p, \) and \( \lambda_p \) are the Calvo parameter, the degree of indexation, the curvature parameter in the aggregator for prices, and the mark-up, and \( \zeta_w, \nu_w, \epsilon_w, \) and \( \lambda_w \) are the corresponding parameters for wages. The resource constraint is

\[ y_t = \frac{c_t}{y_t} c_t + \frac{i_s}{y_t} i_t + \frac{r_k}{y_t} k_t u_t. \tag{39} \]
The policy rule of the monetary authority for the nominal interest rate is policy rule
\[
R_t = \rho_R R_{t-1} + (1 - \rho_R) \left( \psi_1(\chi_t) (\pi_t - \pi_t^*) + \psi_2(\chi_t) (y_t - y_f^t) \right),
\]
where \( \rho_R \) measures the persistence of the policy and the \( \psi \) parameters the sensitivity of the central bank to the fundamentals. In the model without financial frictions, the arbitrage condition makes equal the return to capital and the riskless rate, that is
\[
\frac{r^k_s}{r^k_s + (1 - \delta)} E_t \left[ r^k_{t+1} \right] + \frac{1 - \delta}{r^k_s + (1 - \delta)} E_t \left[ q^k_t \right] - q^k_t = R_t - E_t [\pi_{t+1}] .
\]
(40)
This equation is no longer valid in the model in the context of financial frictions, and then it is replaced by
\[
E_t \left[ \tilde{R}^k_{t+1} - R_t \right] = \zeta_{sp,b}(\chi_t) (q^k_t + \tilde{K}_t - n_t) + \zeta_{sp,\sigma}(\chi_t) \sigma_{w,t},
\]
(41)
and
\[
\tilde{R}^k_t - \pi_t = \frac{r^k_s}{r^k_s + (1 - \delta)} r^k_t + \frac{1 - \delta}{r^k_s + (1 - \delta)} q^k_t - q^k_{t-1}, \tag{42}
\]
where \( \tilde{R}^k_{t+1} \) and \( R_t \) are the gross nominal return on capital for entrepreneurs and the nominal interest rate in the economy, respectively, \( q^k_t \) the price of capital, \( n_t \) the net worth of the entrepreneurs, and \( \sigma_{w,t} \) the uncertainty shock. The law of motion of the entrepreneurial net worth is:
\[
n_t = \zeta_{n,\tilde{R}^k}(\chi_t) \left( \tilde{R}^k_t - \pi_t \right) - \zeta_{n,R}(\chi_t) (R_{t-1} - \pi_t) + \zeta_{n,qK}(\chi_t) \left( q^k_{t-1} + \tilde{K}_{t-1} \right) + \zeta_{n,n}(\chi_t) n_{t-1} - \zeta_{n,\sigma}(\chi_t) \sigma_{w,t-1} .
\]
(43)
It is worth mentioning that \( \zeta_{\cdot}(\chi_t) \) are not structural parameters, but rather the combination of several structural parameters and steady-state values of endogenous variables.

Finally, uncertainty shocks evolve according to
\[
\log \sigma_{w,t} = (1 - \rho_{\sigma_w}(\chi_t)) \log \sigma_w + \rho_{\sigma_w}(\chi_t) \log \sigma_{w,t-1} + \varepsilon_{w,t},
\]
(44)
where \( \rho_{\sigma_w}(\chi_t) \) is the degree of persistence of uncertainty shocks in the regime \( \chi_t \) and \( \varepsilon_{w,t} \) follows the following distribution:
\[
E(\varepsilon_{w,t}) = \text{normal}(\varepsilon_{w,t} | 0, \sigma_{\sigma_w}).
\]
(45)
References


